Fractional Order Derivative Filter Design Using Fourier Transform Interpolation Method

Parvinder[1], Yogesh Juneja[2]

Department of Electronics and communication[1,2]

PDM college of Engg.[1, 2], Haryana

Email: prvndr.shokeen@gmail.com[1], yogeshjunejaer @gmail.com [2]

Abstract— Fractional order derivative is one of the important techniques for the designing of fractional order filter such as differentiator and integrator. Three basic approaches that are used for the design of the fractional order differentiator namely: discretization method, interpolation method and approximation method. All these method provide a great degree of approximation to the designed filter such that it approximates the ideal filter response. Here, in this paper interpolation method for the designing of fractional filter is presented. Different interpolation techniques are presented and studied.

Keywords— DFT, FFT, Interpolation

1. INTRODUCTION

The concept of fractional order derivative can be found in extensive range of many different subject areas [1]-[3]. For this reason, the concept of fractional order derivative should be examined. Fractional derivative can be classified as fractional delay filter and fractional order filter. A fractional delay filter is a filter of digital type having as main function to delay the processed input signal a fractional of the sampling period time. There are several applications where such signal delay value is required, examples of such systems are: timing adjustment in all-digital receivers (symbol synchronization), conversion between arbitrary sampling frequencies, echo cancellation, speech coding and synthesis and musical instruments modelling.

In order to achieve the fractional delay filter function, two main frequency-domain specifications must be met by the filter. The filter magnitude frequency response must have an all-pass behaviour in a wide frequency range, as well as its phase frequency response must be linear with a fixed fractional slope through the bandwidth [2]. Several FIR design methods have been reported during the last two decades.

There are two main design approaches:

1. Time-domain
2. Frequency-domain design methods.

In first one, the fractional delay filter coefficients are easily obtained through classical mathematical interpolation formulas, but there is a small flexibility to meet frequency domain specifications[4].

On the other hand, the frequency-domain methods are based on frequency optimization process, and a more frequency specification control is available. One important result of frequency-domain methods is a highly efficient implementation structure called Farrow structure, which allows online fractional value update.

2. FRACTIONAL ORDER DIFFERENTIATOR

In what follows, let us use the DFT interpolation method and Grünwald-Letnikov derivative, to design a digital fractional order differentiator that approximates the following frequency domain specification as well as possible:

$$H_d(\omega) = (j\omega)^{\upsilon} e^{-j\omega I} \quad ...(1)$$

where I is a prescribed delay value. First, let us define coefficients $a(k)$ below

$$a(k) = (-1)^k C_k^{\upsilon} \quad ...(2)$$

then the fractional derivative in Eq.(5.1) can be rewritten as

$$D^{\upsilon} s(t) = \lim_{h \to 0} \sum_{k=0}^{\infty} \frac{a(k)}{h^{\upsilon}} s(t-kh) \quad ...(3)$$

It is clear that the $a(k)$ is a rapidly decaying sequence for various order υ. Thus, by truncation, $D^{\upsilon}s(t)$ in can be approximated by

$$D^{\upsilon} s(t) \approx \lim_{h \to 0} \sum_{k=0}^{K} \frac{a(k)}{h^{\upsilon}} s(t-kh) \quad ...(4)$$

where K is truncation order. Moreover, by removing limit, the $D^{\upsilon}s(t)$ can be further approximated by

$$D^{\upsilon}s(t) \approx \sum_{k=0}^{K} \frac{a(k)}{h^{\upsilon}} s(t-kh) \qquad ...(5)$$

Obviously, the smaller h is, the better approximation. By taking $t = n - I$, the discrete-time derivative signal $D^{v} s(n - I)$ can be obtained as

$$D^{\upsilon}s(n-I) \approx \sum_{k=0}^{K} \frac{a(k)}{h^{\upsilon}} s(n-I-kh) \qquad ...(6)$$

Because $s(n - I- kh)$ are non-integer delay samples of signal $s(n)$, the $s(n - I- kh)$ needs to be estimated by using the formula in

$$s(n-I-kh) = \sum_{r=0}^{N-1} w(r, I+kh) s(n-r) \qquad ...(7)$$

Substituting Eq.(6) into Eq.(7), we have

$$D^{\upsilon}s(n-I) \approx \sum_{k=0}^{K} \frac{a(k)}{h^{\upsilon}} \sum_{r=0}^{N-1} w(r, I+kh) s(n-r)$$
$$= \sum_{r=0}^{N-1} \left[\frac{1}{h^{\upsilon}} \sum_{k=0}^{K} a(k) w(r, I+kh) \right] s(n-r) \qquad ...(8)$$

Defining coefficient

$$g(r) = \frac{1}{h^{\upsilon}} \sum_{k=0}^{K} a(k) w(r, I+kh) \qquad ...(9)$$

then Eq.(8) can be rewritten as the following convolution form:

$$D^{\upsilon}s(n-I) \approx \sum_{r=0}^{N-1} g(r) s(n-r)$$
$$= g(n) * s(n) \qquad ...(10)$$

where * denotes the convolution sum operator. Taking z transform at both sides of Eq.(10), we get

$$Y(z) = \left(\sum_{r=0}^{N-1} g(r) z^{-r} \right) S(z) \qquad ...(11)$$

where $Y(z)$ is z-transform of $D^{v}s(n - I)$ and $S(z)$ is z transform of $s(n)$. Let FIR filter be defined as

$$G(z) = \sum_{r=0}^{N-1} g(r) z^{-r} \qquad ...(12)$$

then $G(z)$ is the transfer function of the designed fractional order differentiator which will approximate ideal frequency response $(jw)^{v}\, e^{(-jwI)}$ well [5]-[7].

3. SIMULATION RESULT

The designs of fractional order differentiator are presented using the discrete Fourier transform interpolation techniques. Simulation has been done in MATLAB and signal processing toolbox of matlab.

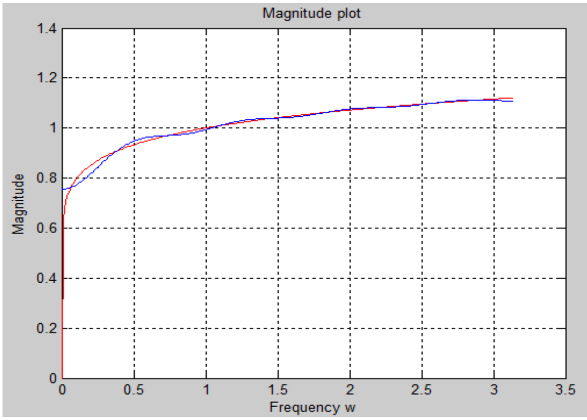

Fig. 1 Magnitude response of design fractional order differentiator based on discrete Fourier transform interpolation technique with order v=0.1.

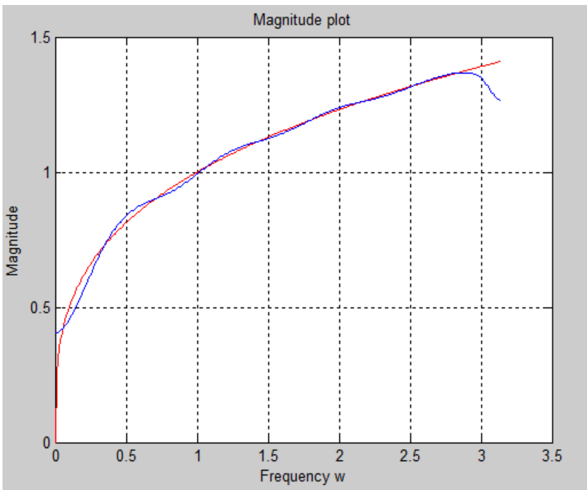

Fig. 2 Magnitude response of design fractional order differentiator based on discrete Fourier transform interpolation technique with order v=0.3.

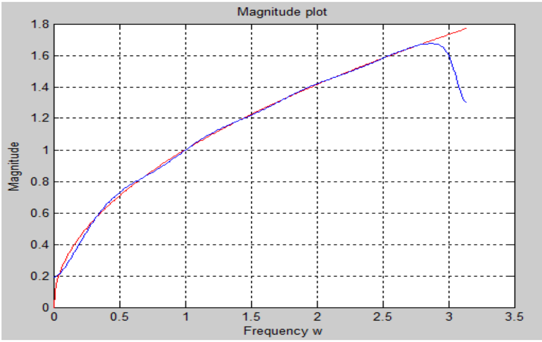

Fig. 3 Magnitude response of design fractional order differentiator based on discrete Fourier transform interpolation technique with order v=0.5.

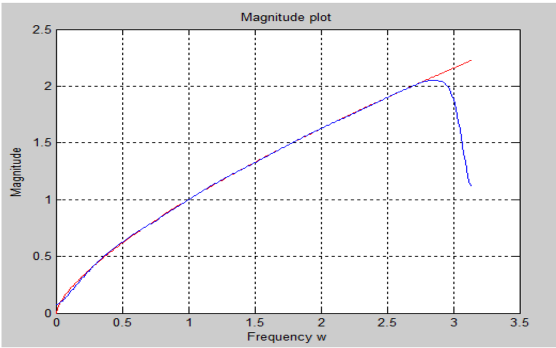

Fig. 4 Magnitude response of design fractional order differentiator based on discrete Fourier transform interpolation technique with order v=0.7.

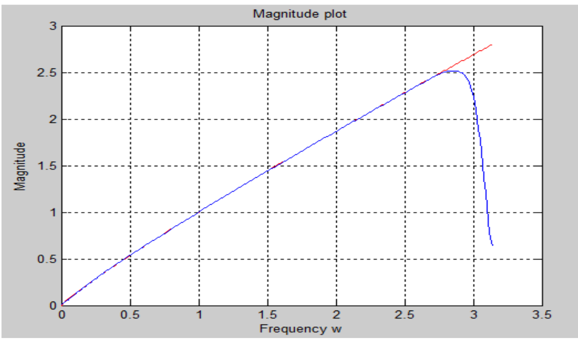

Fig. 5 Magnitude response of design fractional order differentiator based on discrete Fourier transform interpolation technique with order v=0.9

4. CONCLUSION

In this paper, discrete Fourier transform interpolation technique for the designing of fractional order differentiator is used. The coefficient of the design filter is efficiently calculate using DFT interpolation. Magnitude Response is to be consider for the design of fractional order differentiator. Here, magnitude response is considered as the performance measure criteria to be chosen.

REFERENCES

[1]. J. Vesma and T. Saramäki, “Optimization and efficient implementation of FIR filters with adjustable fractional delay,” in Proc. IEEE Int. Symp. Circuits & Syst.,Hong Kong, June 1997, pp. 2256-2259.

[2]. T. Saramäki and M. Ritoniemi, "An efficient approach for conversion between arbitrary sampling frequencies," in Proc. IEEE Int. Symp. Circuits & Syst., Atlanta,GA, May 1996, pp. 285-288.

[3]. Nitin Kumar,Brij Mohan,Sovit Attri and Sana Farheen “Design of Fractional Order Digital Differentiator Using RBF Interpolation Method ” International Journal of Digital Signal and Image Processing (IJDSIP)Vol. 2, No. 1(March 2014)

[4]. Zöhre Kara Kayikci, and Günther Palm “Word Recognition and Learning based on Associative Memories and Hidden Markov Models” World Academy of Science, Engineering and Technology Vol:2 2008-01-28

[5]. L. B. Almeida, "Product and convolution theorems for the fractional Fourier transform", IEEE Trans. Signal Processing Letters, vol. 4, pp.15 -17 1997.

[6]. J. Vesma and T. Saramäki, “Interpolation filters with arbitrary frequency response for all-digital receivers,” in Proc. IEEE Int. Symp. Circuits & Syst., Atlanta, GA, May 1996, pp. 568-571.

[7]. J. Vesma, M. Renfors, and J. Rinne, “Comparison Of efficient interpolation techniques for symbol timing recovery,” in Proc. IEEE Globecom 96, London, UK, Nov.1996, pp. 953-957.